

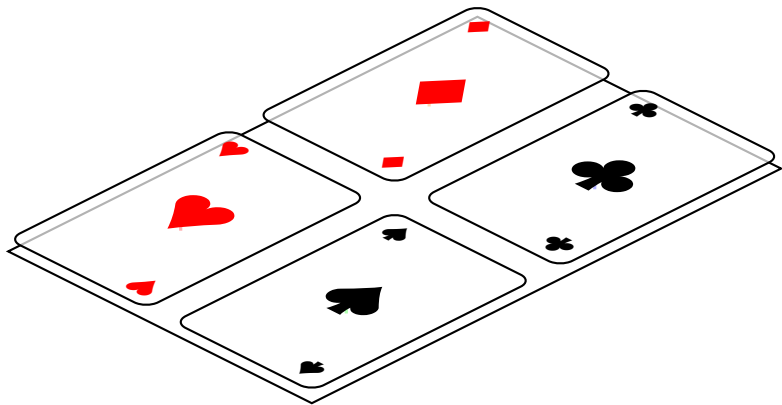
Random Interchange Process

Peter Mühlbacher

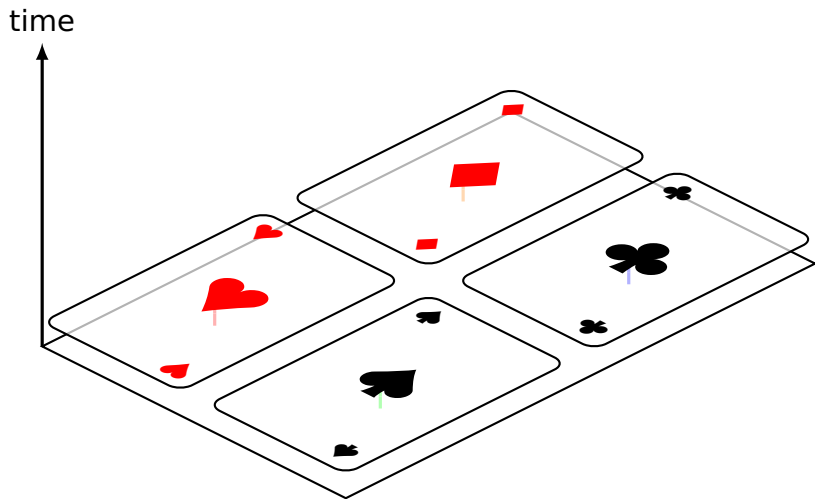
University of Warwick

June 12, 2019

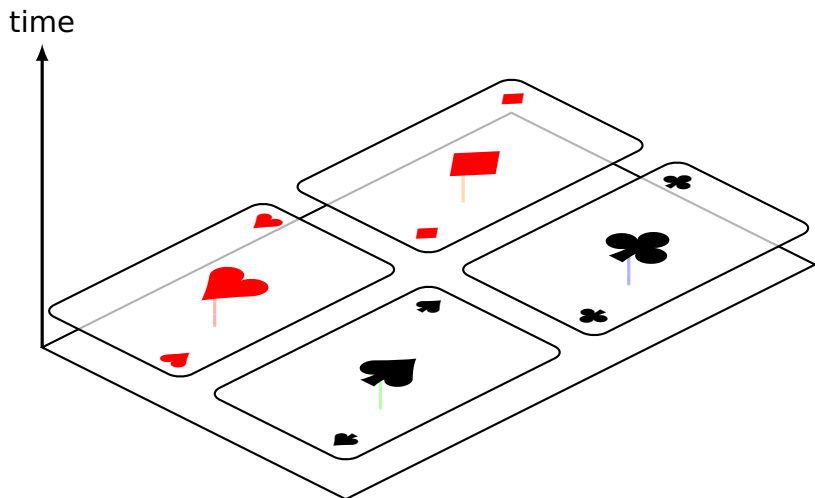
A concrete example: Shuffling adjacent cards



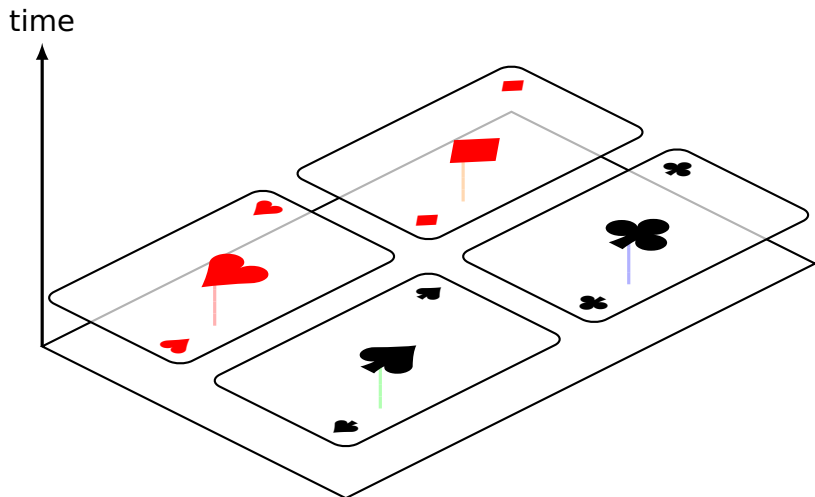
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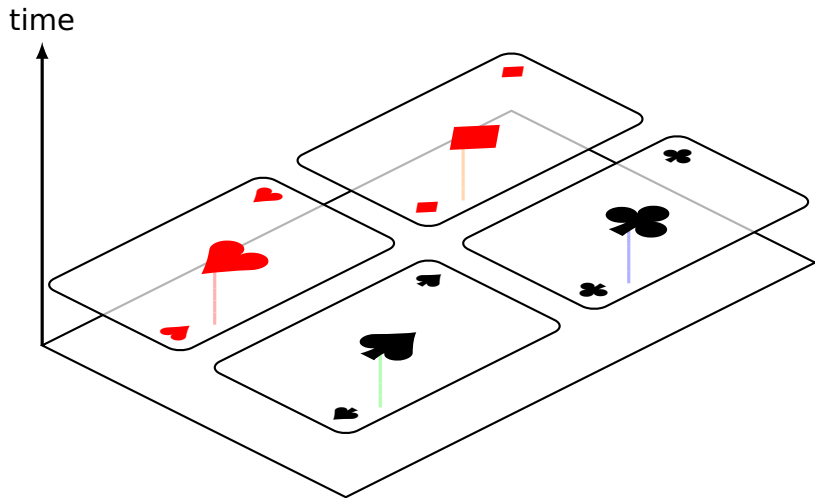
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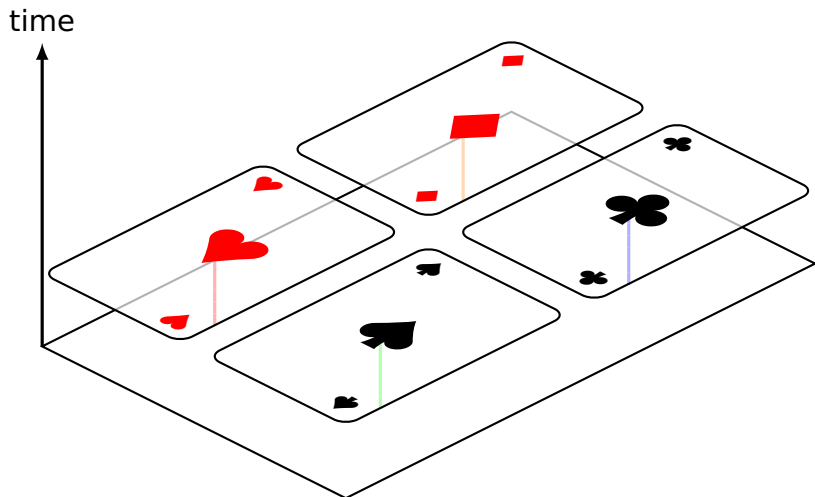
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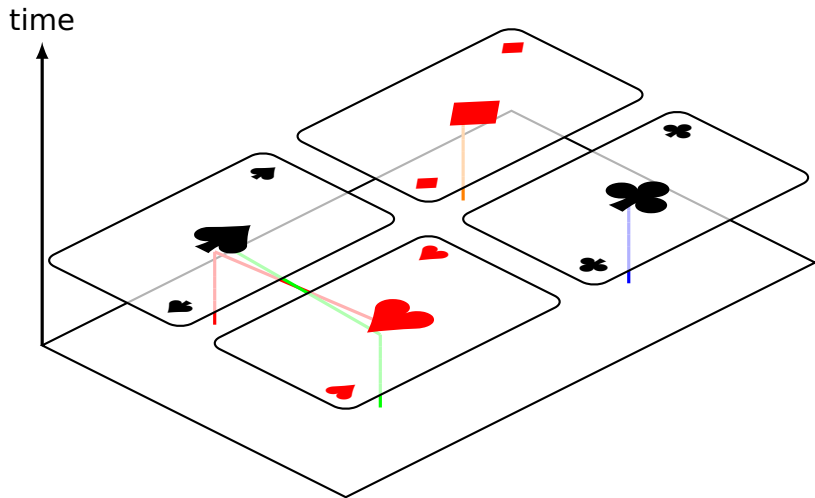
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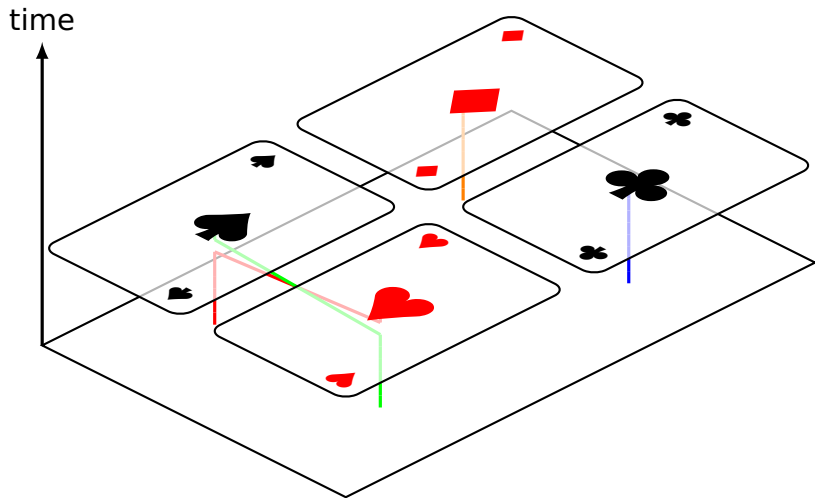
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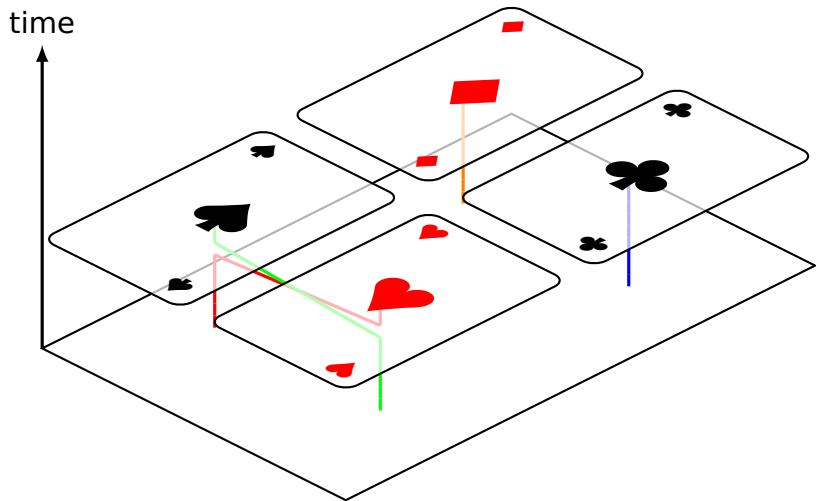
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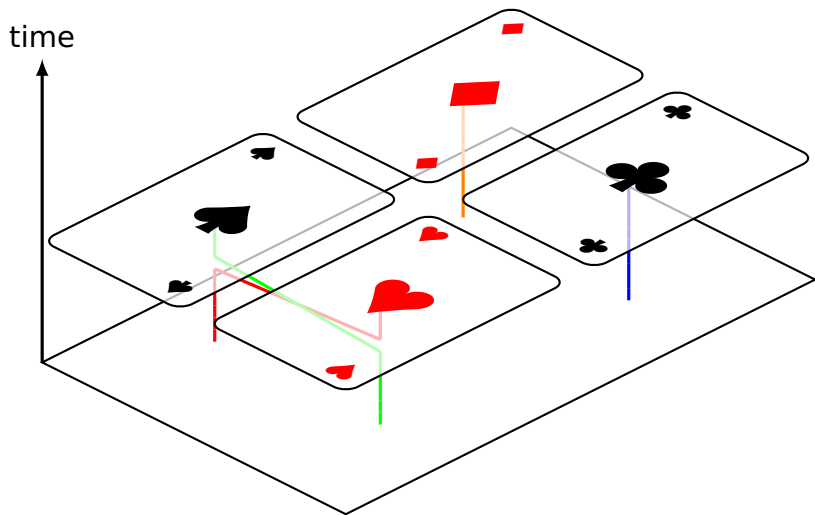
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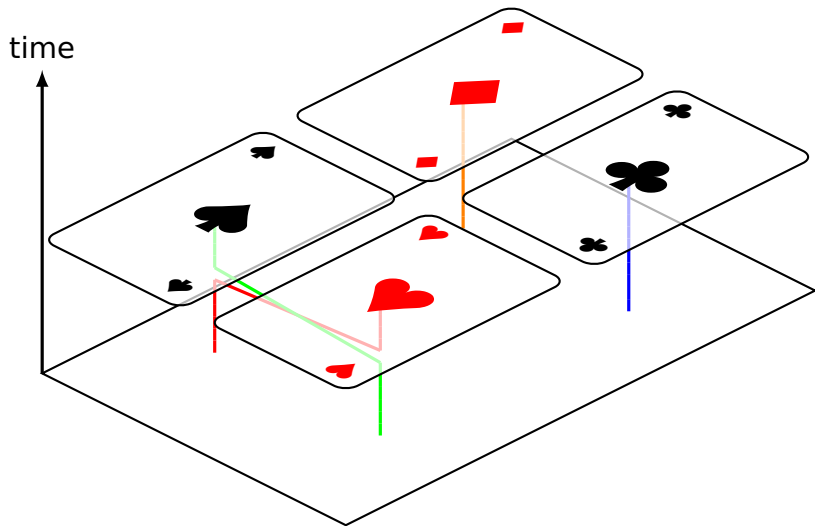
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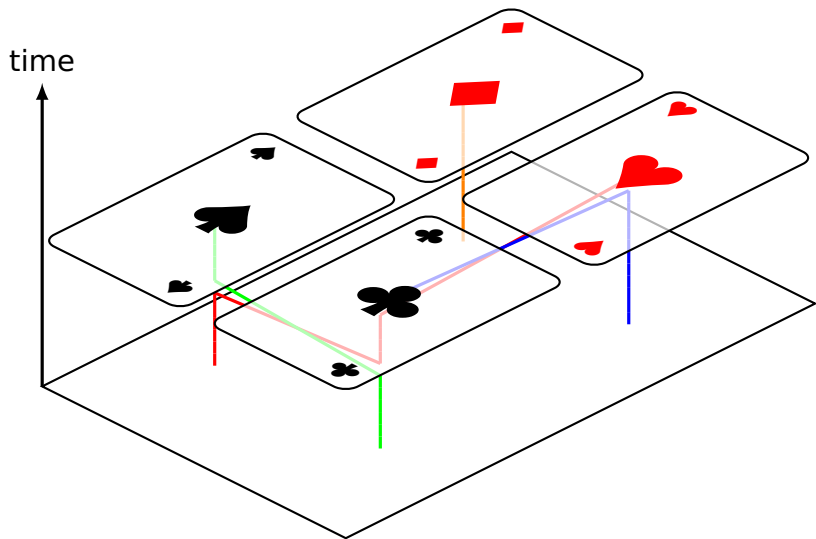
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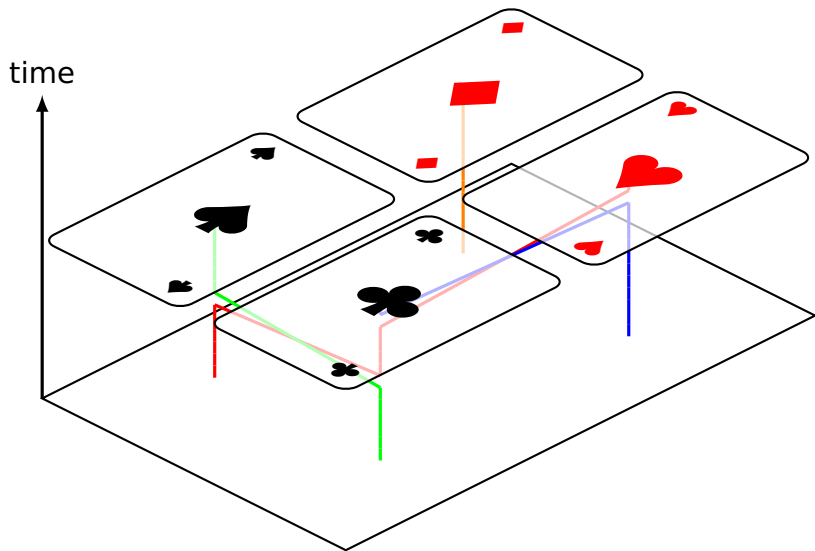
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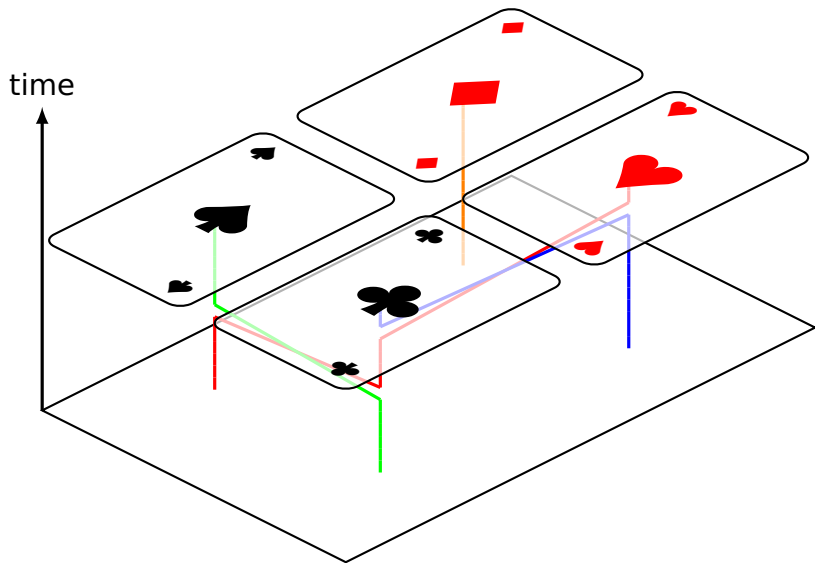
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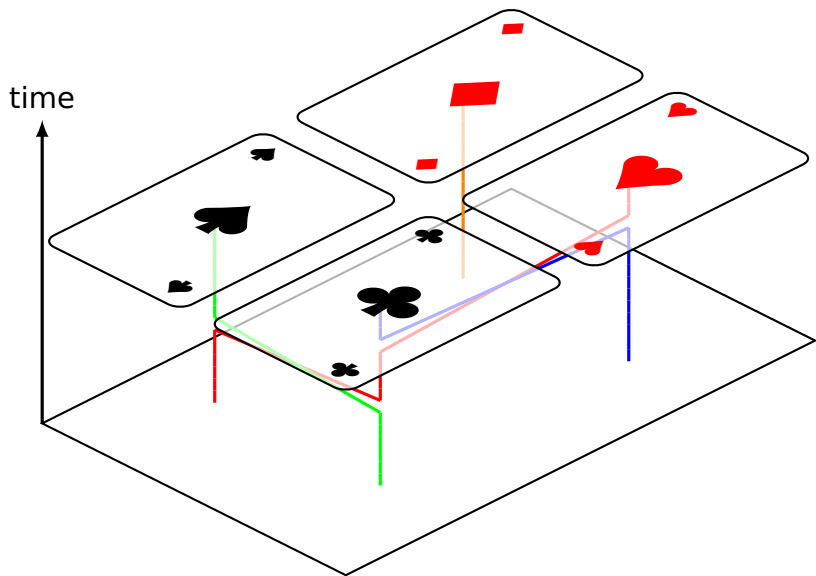
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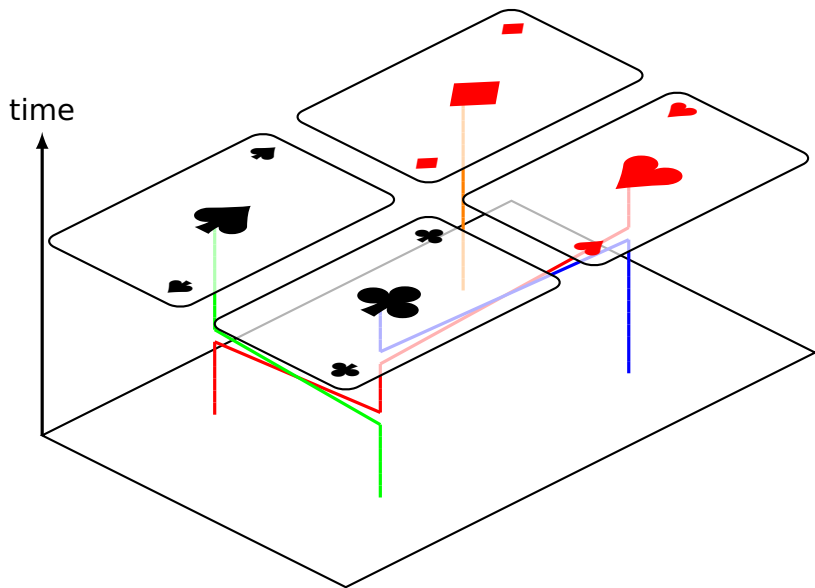
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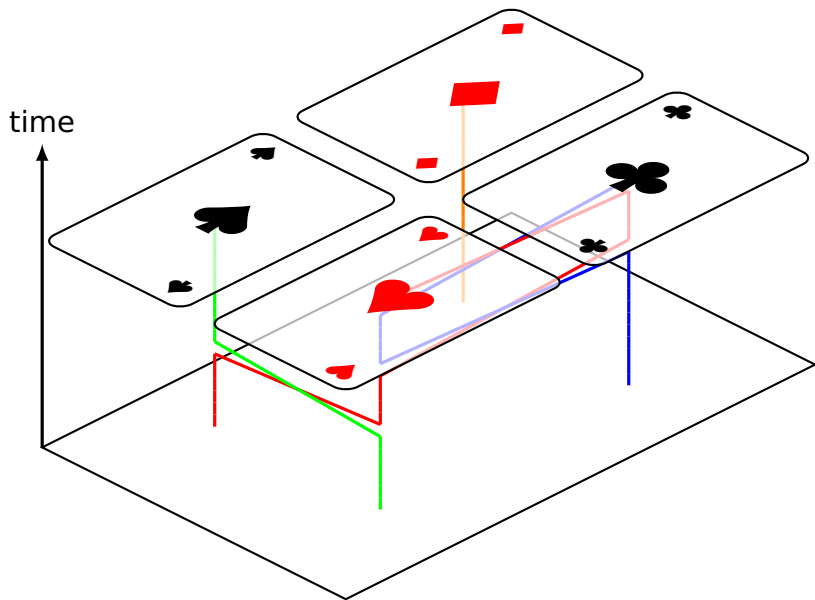
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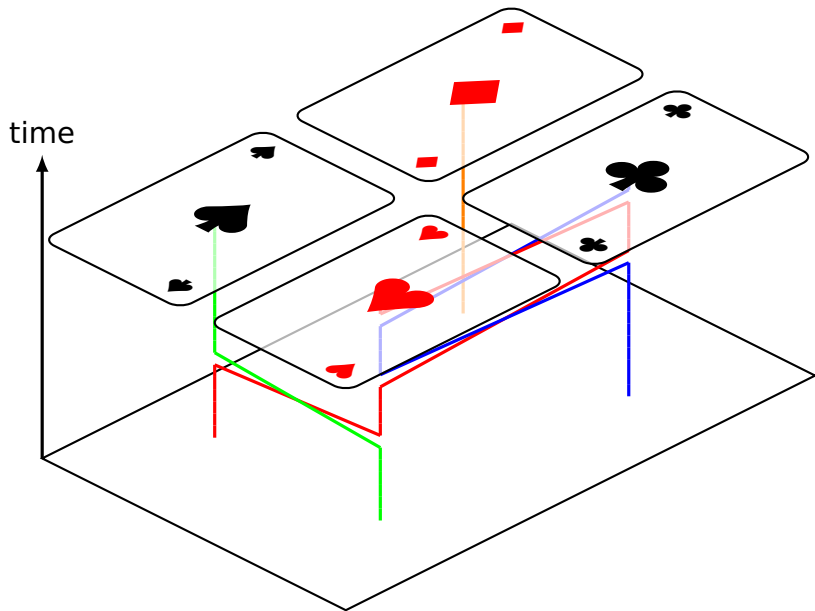
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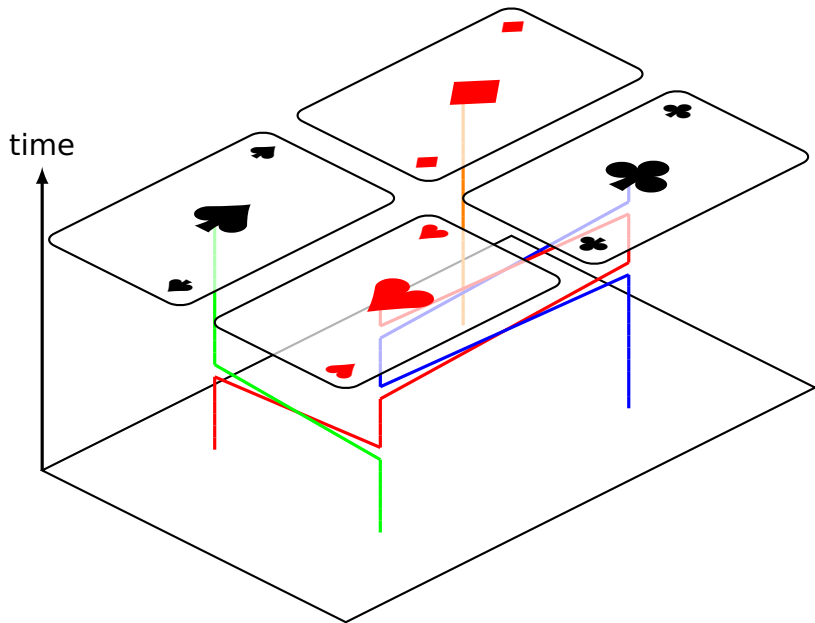
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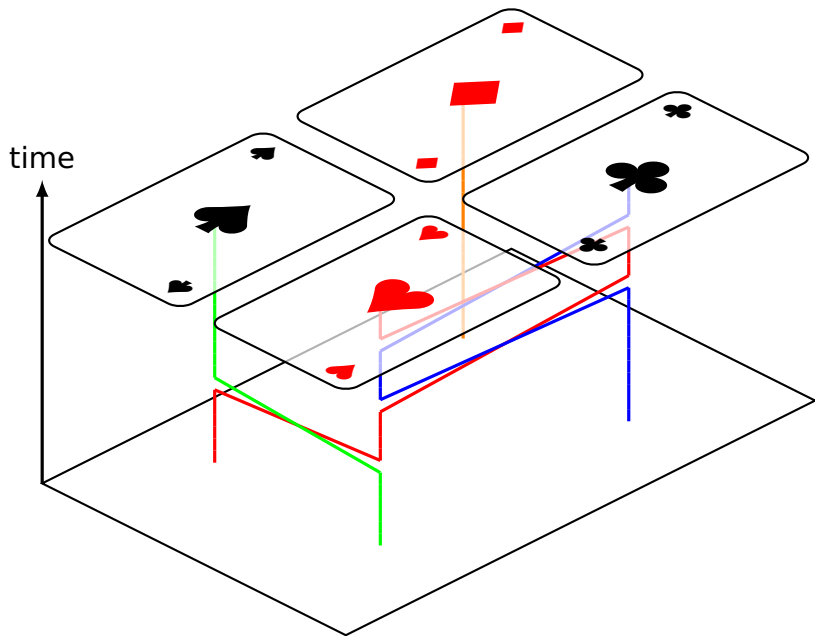
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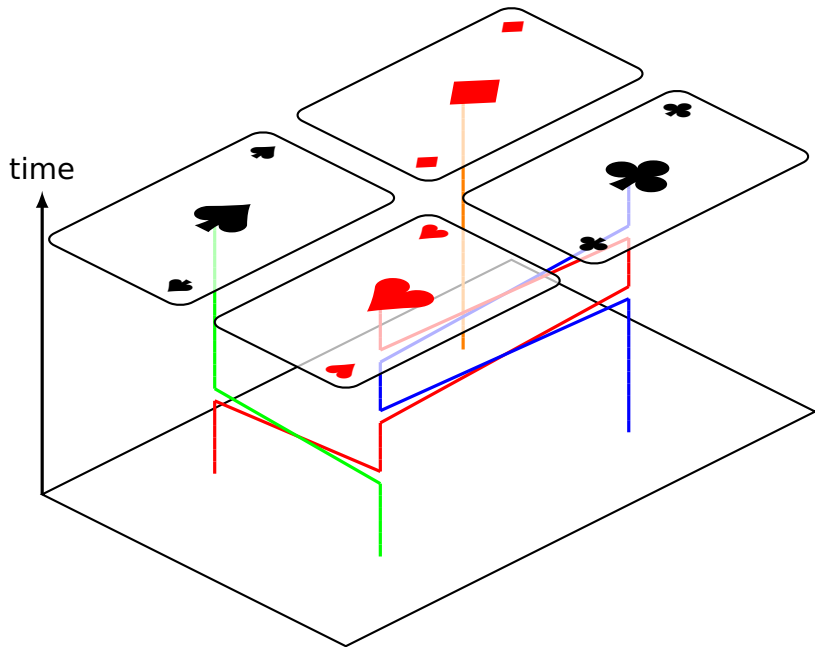
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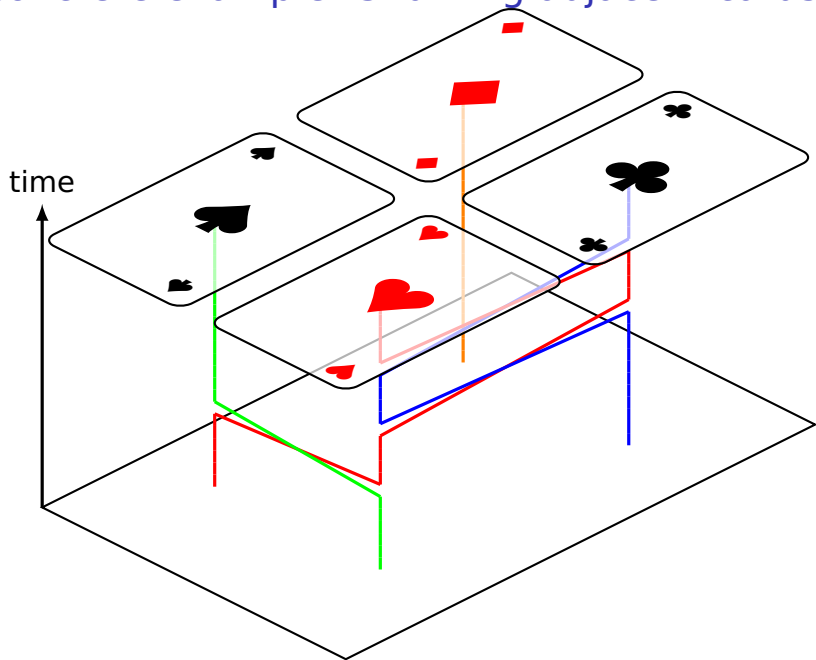
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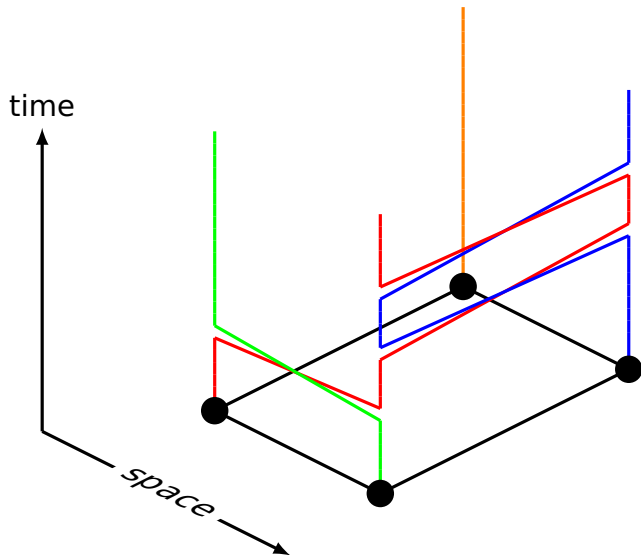
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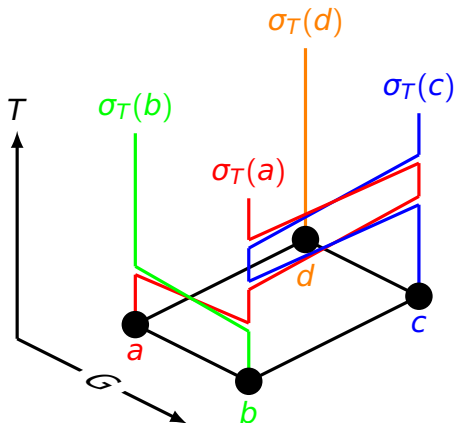
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Definition of the random interchange model

Fix some $T \in (0, \infty)$ and a graph $G = (V, E)$ (e.g. $\mathbb{Z}^d$);
on each edge $e \in E$ place $\text{Poisson}(T)$ crosses at
independent, uniform random heights in $[0, T)$.

$\Rightarrow$ get random permutation σ_T :

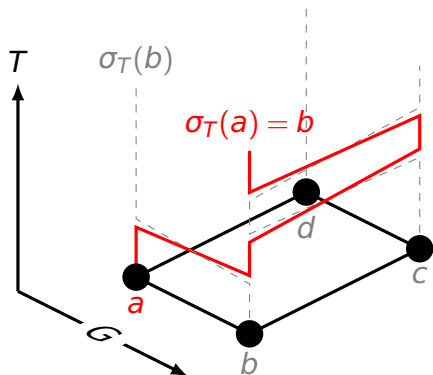


$\sigma_T =$ product of transpositions supported on edges of G !

What's the point?

This representation lets us relate random permutations to

- ▶ **random walk models** $\Rightarrow$ analyse cycle structure of σ_T
- ▶ percolation $\Rightarrow$ upper bound on large cycles

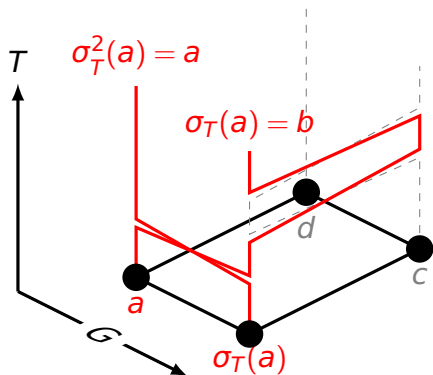


- ▶ Fix any vertex, say a .
- ▶ Cycle with a is $(a, \sigma_T(a), \sigma_T^2(a), \dots)$.

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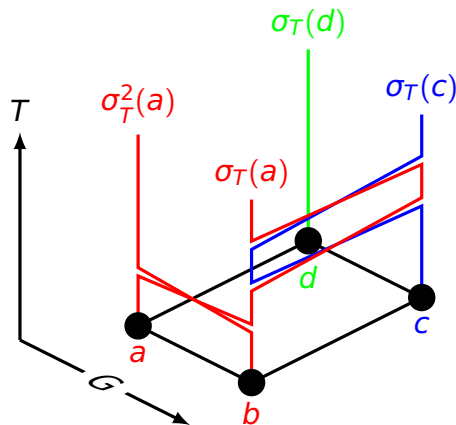


- ▶ Fix any vertex, say a .
- ▶ Cycle with a is $(a, \sigma_T(a), \sigma_T^2(a), \dots)$.
- ▶ Strongly dependent random walks on G !
 $\Rightarrow$ It's **really** hard.

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This representation lets us relate random permutations to

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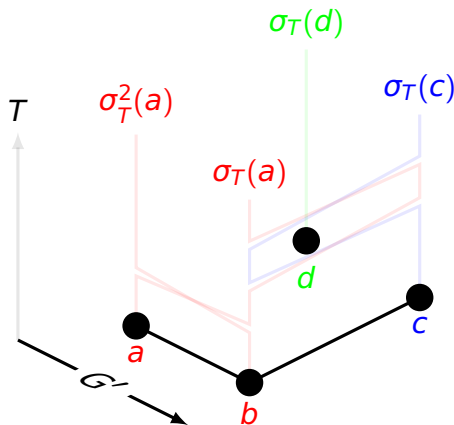


- ▶ Fix any two vertices, say a, d .
- ▶ Are they in the same cycle?

What's the point?

This representation lets us relate random permutations to

- ▶ random walk models $\Rightarrow$ analyse cycle structure of σ_T
- ▶ **percolation** $\Rightarrow$ upper bound on large cycles



- ▶ Fix any two vertices, say a , d .
- ▶ Are they in the same cycle?
- ▶ Construct G' by keeping only edges with ≥ 1 crosses.
- ▶ **Necessary** condition: There is a path in G' from a to d .

Open questions

- (Q1) Is there a T such that there are infinite connected components of G' , but no infinite cycles?
- (Q2) Does increasing T always increase the probability of finding infinite cycles?
- (Q3) Is there a T such that there are infinite cycles?

Partial progress towards (Q1):

For various graphs of unbounded degree (e.g. the complete graph) the answer to (Q1) is **no**.

Theorem (M., '18)

*On graphs of bounded degree (e.g. $\mathbb{Z}^d$) the answer to (Q1) is **yes**.*

Conclusion

Progress has been made using techniques from

- ▶ probability theory,
 - ▶ percolation theory,
 - ▶ (non-Markovian) random walks,
 - ▶ mixing times of Markov chains,
- ▶ geometry (of groups),
- ▶ representation theory,
- ▶ quantum spin systems.

If you are familiar with any of that and are interested what the above questions imply for your field or have some ideas, please talk to me!

Thank you for your attention!